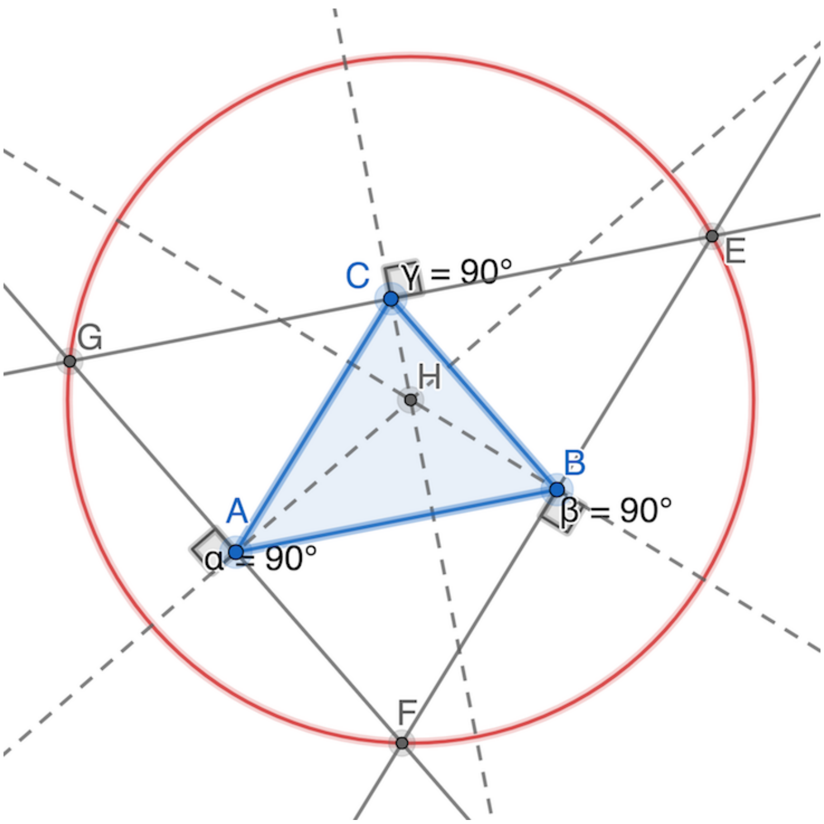
Orthocentre

Theorem

The altitudes of any triangle are concurrent in the orthocentre H,



Proof.

Triangle ABC was drawn at random.
The dotted lines are the altitudes of Triangle ABC.
Where these lines meet is called the Orthocentre, represented by Point H.
Triangle EFG was constucted by lines parallel to sides on Triangle ABC.
AB and EG are parallel. BC and FG are parallel. AC and EF are parallel.
The dotted lines are also the perpendicular bisectors of Triangle ABC.
Where the dotted lines meet is also the Circumcentre of Triangle EFG.
This is shown by the red circle.
The red circle goes through all vertices of EFG.
The red circle also has Point H as its circumcentre. □

[The above diagram, and sentences of the proof, are copied from Leon's own work in Geogebra.]

Note. There are other proofs of the concurrency of the altitudes of a triangle, but the above method is rather nice. The property of being concurrent of the altitudes of triangle ABC is just 'seen' to be inherited from the (already proven) concurrency of the perpendicular bisectors of triangle EGF. The altitudes of ABC are the very same lines that are the bisectors of EGF.

In the next result (Euler Line) we make use of the fact that triangle EGF is similar to, and twice the size of, triangle ABC. This is because ABCG and ABEC are each parallelograms. Their opposite sides are equal, so $GC = AB = CE$, and thus $GE = 2AB$. Similarly for the other parallel sides.