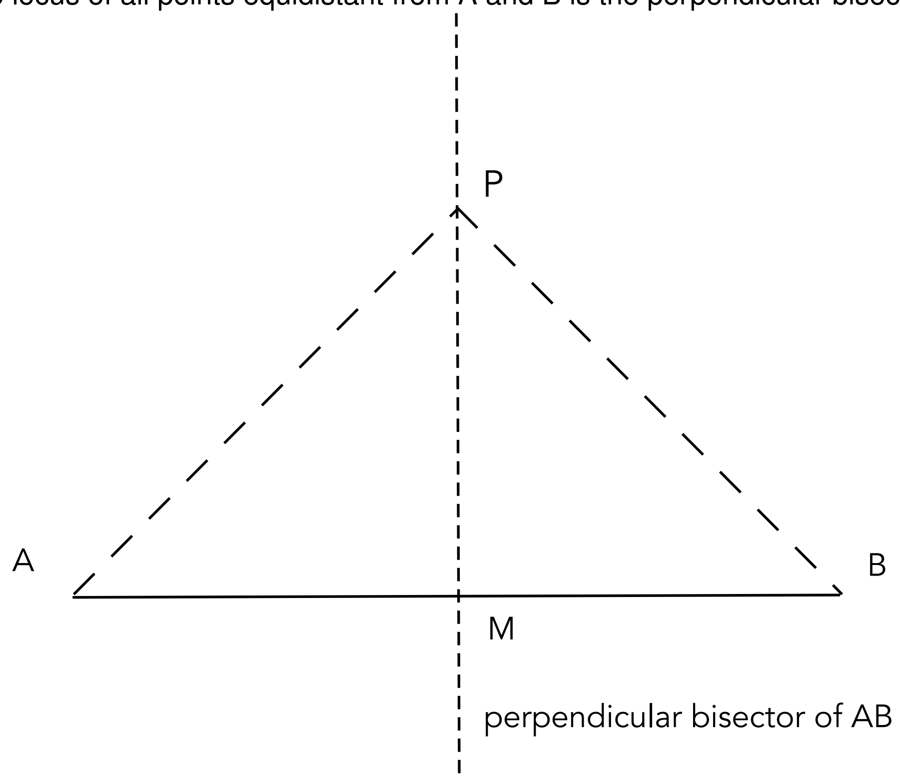
Circumcentre

Lemma 1.

The locus of all points equidistant from A and B is the perpendicular bisector of the line AB.



Proof.

Let P be any point on the perpendicular bisector of AB.

In $\triangle$s AMP, BMP

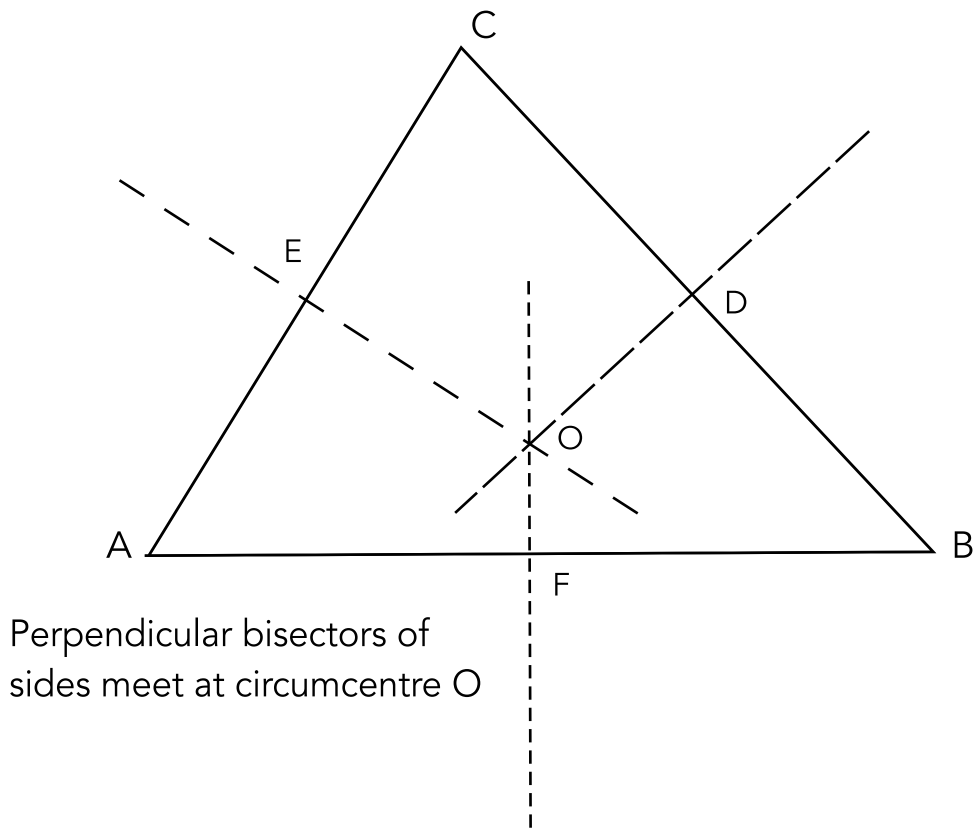
AM = MB
$\angle$ AMP = $\angle$ BMP $(= 90°)$
PM is common

So $\triangle$s AMP, BMP are congruent (SAS), and hence PA = PB.
If P were not on the perpendicular bisector of AB, the above triangles are not congruent and PA $\neq$ PB. □

(Continued)

Theorem.

The perpendicular bisectors of the sides of a triangle are concurrent.



Proof.

Let bisectors at D and E meet at O.
By Lemma 1 OB = OC and OC = OA, therefore OB = OA.
So, again by Lemma 1, we have that O lies on the perpendicular bisector of AB.
Thus all three perpendicular bisectors are concurrent in O, the circumcentre of $\triangle$ ABC. □