

[Scene:

*It's autumn 1972 and Mr Leonard John has started as a new Maths teacher at the Tree House Grammar School where he is enjoying the start of his new job.*

*The bell has just gone for the morning break and the Head of Maths (Dr Rigorous) has planned to catch up today with how the new maths teachers who started a month ago are settling in. He's a little worried about what he has heard about an illustration of Pythagoras' Theorem (made with pieces of coloured cardboard!) being described in Mr John's lessons as a 'proof'. He likes to be thought of as the 'friendly leader' so he uses first names with all his staff – unusual at this time. He approaches Mr John ...]*

Rigorous: Ah, Leonard, how are you getting on? The boys like you - I've heard from young Braithwaite that you've made quite a hit with 2 Set3! Get your coffee and come and let's sit by the fire.

John: Yes, that Set 3 are a nice lot - they were as good as gold for the first week but now they have become rather lively and it's a hard job to deal with all their questions as well cover the work I had planned for the lesson.

Rigorous: Good they are asking questions, but be on the look out, some of them may just be finding out the best ways of distracting you from the work you really want them to do.

John: It's difficult when they ask about something I'm really keen on, they liked a nice visual proof of Pythagoras they showed us in the PGCE. I was bowled over by it ... much clearer than the old Euclid proof with congruent triangles.

Rigorous: Yes, I heard about your 'red and blue' Pythagoras ... but, you know, I don't think we should call such an illustration a proper 'proof'.

John: Oh dear! Why not? They told us in the training sessions it was a proof. And most of us students thought it was far more convincing than the long-winded method of Euclid with all his extra construction lines.

Rigorous: Sounds like you remember the Euclid proof quite well. There are three objections to a visual proof - such as the one, I think, you showed them. First it's a kind of 'category mistake' as the philosophers call it. Pythagoras theorem comes in Book 1 of Euclid's Elements where a point is defined as 'that which has no part', and a line has no breadth - it's a world of the mind and not the normal physical world. Lines drawn with a pen, and in colours, are physical and can help us *\*imagine\** geometry but are not the real objects of geometry.

John: So you mean a visual proof is a bit like a **poem** about daffodils and trees rather than the real things?

Rigorous: That's an interesting comparison but I wouldn't want to press the analogy too far.

John: My arty friends tell me poetry and painting can express some truths better than prose.

R: Maybe not mathematical truths. These are deep things we should come back to. But to mention two further objections: Pythagoras theorem is general, or universal, it applies to *\*all\** right-angled triangles, a drawing or a cardboard instance is only one particular triangle. It only *\*suggests\** other triangles. And finally a theorem (like Pythagoras) is *\*exact\** and for sure drawn lines always have some breadth, two card triangles will always have a small gap between them, and so on. They are never exact.

J: Oh dear, the boys of Set 3 will be disappointed if I tell them that wasn't a proof after all!

R: Proof is such an important concept in maths we have to be careful about it. You can tell the class that some examiners (at the 'O'level) would not like a visual proof of Pythagoras - they might give few (or no) marks for it. And imagine trying to describe all those movements of triangles by writing sentences!

*[Bell rings again for the end of break.]*

J: Well, you have given me a lot to think about.

R: In the end you have to make up your own mind on this sort of thing. The boys will tell immediately if you don't really believe what you are saying.

J: Can I talk with you more about this tomorrow?

R: Yes, of course, I'll be glad to ... until tomorrow's morning break ... *[They go to classes.]*

*[It's break time the next day and our two teachers are following up yesterday's discussion.]*

J: I've just had Set 3 last lesson - one of them asked why they can't do all their geometry proofs by using pictures like we did for Pythagoras!

R: Oh dear ...

J: I looked on the internet last night for 'congruence' and found something interesting by a chap called Bolzano. He didn't like congruence.

R: I've heard of him - there are one or two theorems named after him in calculus. But I didn't know he wrote about geometry.

J: Well, it was over 200 years ago so it's probably out of date ... but he said congruence did not fit in well with other ideas in Euclid's Elements - actually he seemed to criticise it like you were criticising physical things ('empirical' I think he called it), and he said it was really not necessary anyway ... I've printed it out for you if you are interested.

*[Gives to Dr Rigorous]*

R: OK, thank you, I shall take a look ... and get back to you. You've given me something to think about now!

*[They are interrupted by another teacher wanting to talk to Dr R.]*