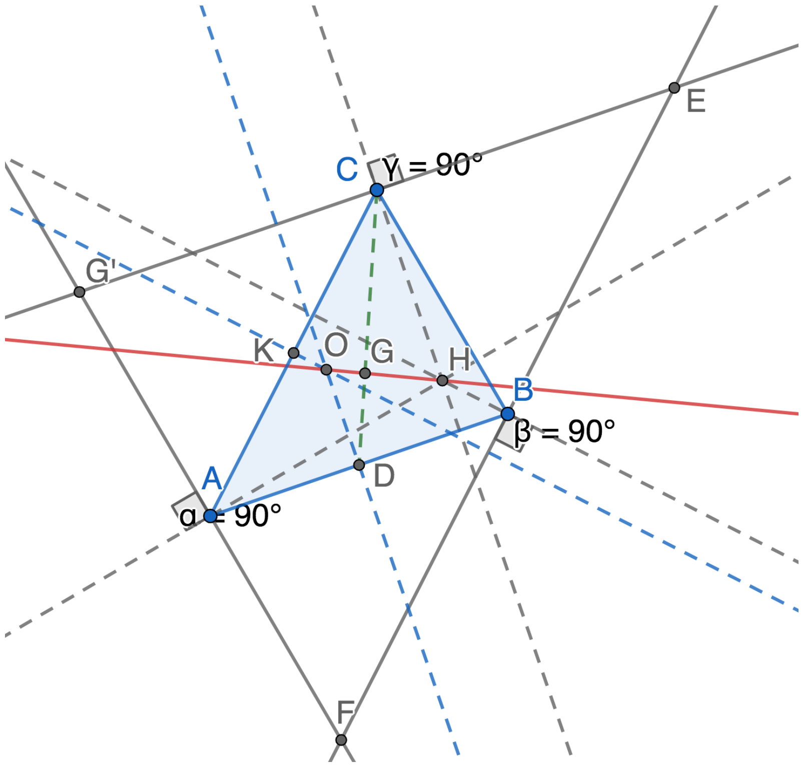
# Euler Line

### Theorem

In any triangle the circumcentre O, the centroid G, and the orthocentre H, are collinear. The line joining them is the Euler line and G divides OH in the ratio of $1:2$.




**Proof.**

Our diagram with blue triangle ABC, and black twice-as-large triangle EFG, is similar to the previous diagram for the orthocentre but now with two additions. The blue dashted lines are perpendicular bisectors through midpoints D and K meeting in circumcentre O, the green dashed line is a median CD. Let OH meet CD in G. We show that G is the centroid of △ ABC. As noted above △s ABC, EGF are similar, so all corresponding lengths in the triangles are in ratio $1:2$. In particular H is the circumcentre of △ EGF, so $CH = 2DO$. In △s DGO, CGH the angles at G are equal vertically opposite angles, and the angles at G are equal alternate angles, so the triangles are similar and $CG = 2GD$. So G is the centroid of △ ABC. Also we see that G divides OH in the ratio $1:2$. □

(Continued)

**And a little more ... !** There is even more collinearity. Something called the nine point circle exists for every triangle and passes through the feet of the altitudes, the midpoints of the sides, and the midpoints of lines joining each vertex to H. The centre of the nine point circle, say N, lies at the midpoint of OH. It's easy to prove this and we leave it as an exercise for the reader. So finally $OG : GN : NH = 2 : 1 : 3$.
Below is a diagram of the situation with the triangle ABC in a different orientation but the Euler line is clearly marked showing the ratios concerned.