Short Stories from Mathematics.

Episode 4: Proofs and Fallacies in Mathematics

Tuesday 25th February 7.30 pm **Tree House Bookshop**

WHO NEEDS PROOFS?

When someone tells you something new, or quite surprising, we often say, 'Are you sure?', or 'Can you prove it?'.

Some examples:

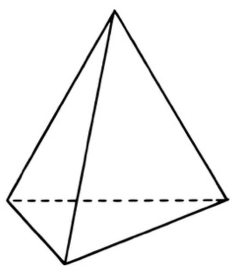
(1) Here's something surprising (if you have not seen it before!). Take a very long rope and wrap it tightly around the equator of the earth, pressing it down over hills and pulling tight across the sea. Cut the rope and insert an extra one metre (just over a yard) of rope so that it goes slack and put in some supports so there is a small gap between the earth and the (now longer) rope. What is the size of this gap?

The answer is about 16cm (or 6inches). Try proving it – it's surprisingly easy. There is no trick in this, it's correct and independent of the size of the 'circle'. Why are we (or at least our intuition) so surprised?

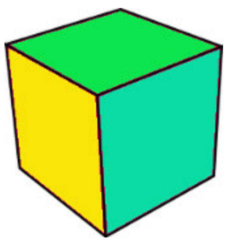
(2) Around 1750 Euler noticed that in the regular solids the number of vertices (V), the number of edges (E) and the number of faces (F), were all connected by the formula,

$V - E + F = 2$. But he could not prove it for any polyhedron. Can you prove it?

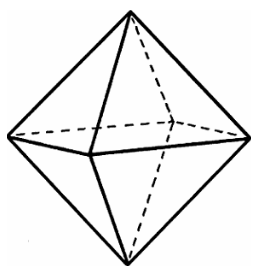
First of all, check it out for yourself with the following solids:



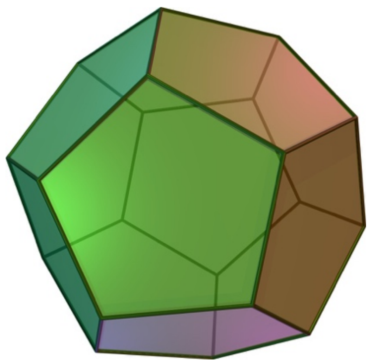
Tetrahedron



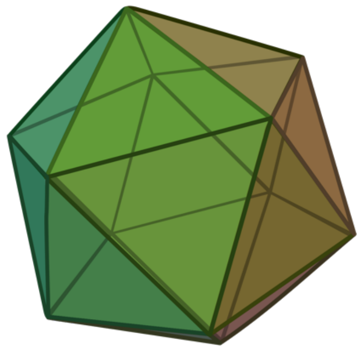
Cube



Octahedron



Dodecahedron (12 pentagons)



Icosahedron (20 triangles)

We shall give a proof given by a famous mathematician (Cauchy) and then show some problems with it ...

(3) Lyness Cycles. A well-known sequence of numbers is the Fibonacci sequence which begins with 1, 1, 2, 3, 5, ... each term being the sum of the previous two terms. Obviously the terms keep getting bigger. Here's a slightly more complicated 'rule' for making a sequence. Begin with two numbers, say, 2, 5, and to make the next we add one to the last (5 in this case) and divide by the one before it, thus making 3. Apply the rule again to get the fraction 4/5. Carry on and you should get a surprise on the 6th and 7th terms.
This might have been just a fluke with our starting numbers. Can you prove the surprise will (nearly) always happen?

Proof is characteristic of mathematics, in the way (roughly) that repeatable experiments are characteristic of natural science. Sound proofs are a necessary (not sufficient) 'passport' for new work to be accepted by the community and by journals - the subject cannot grow without new results and theories being proved. And it grows at an enormous rate.

Most mathematical research is published in refereed research journals. There are well over a thousand such journals, publishing several times a year. That means a lot of proofs!

We shall look at some of the major kinds of proof in mathematics, how fashions have changed what is 'acceptable', and how computers have both helped us and confused us.

On this occasion it will be useful to have a pencil and paper (or equivalent) with you.

For further details of this series, and earlier sessions, see:

www.modellingminds.co.uk/kenilworth-mathematics